

Worked Solutions

Edexcel C3 Paper E

1.
$$\frac{(2x-3)(x+1)}{(x+1)} + \frac{(x-2)(x+2)}{(x+2)} = 3x - 5 \Rightarrow A = 3, B = -5 \quad (4)$$

$$3x - 5 = x^2 - 9 \Rightarrow x^2 - 3x - 4 = 0$$

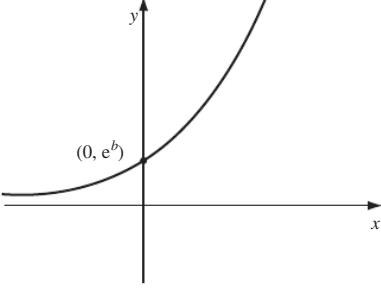
$$(x-4)(x+1) = 0$$

$$\Rightarrow x = +4 \text{ or } x = -1 \quad (2)$$

2. (a) \because domain of $\sin^{-1}(x)$ $\quad \frac{-\pi}{2} < x < 0 \quad \sin x = -\frac{3}{5}$
 $\cos^2 x + \sin^2 x = 1$
 $\therefore \cos^2 x + \frac{9}{25} = 1 \quad \therefore \cos^2 x = \frac{16}{25} \Rightarrow \cos x = +\frac{4}{5} \quad (4)$

(b) $\cos 2x = 1 - 2 \sin^2 x = 1 - \frac{18}{25} = \frac{7}{25} \quad (2)$

3. (a)



A Cartesian coordinate system showing a curve that passes through the point $(0, e^b)$. The curve is increasing and concave up, representing an exponential growth function.

(2)

(b) $e^b = 4 \quad \therefore b = \ln 4 \quad (1)$

(c) $\frac{dy}{dx} = a \cdot e^{ax+b} = a \cdot e^{ax} \cdot e^b = 4ae^{ax}$

 $x = 2, \quad 10e^5 = 4ae^{2a} \quad a = 2\frac{1}{2} \quad (5)$

4. (a) $\frac{d}{dx} (\cos^2 x) = 2 \cos x (-\sin x)$
 $= -2 \cos x \sin x.$

(b) $\frac{d}{dx} \left(\frac{\ln x}{x} \right) = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$
 $= \frac{1 - \ln x}{x^2}$

(c) $\frac{d}{dx} (x^2 e^x) = x^2 \cdot e^x + e^x \cdot 2x$

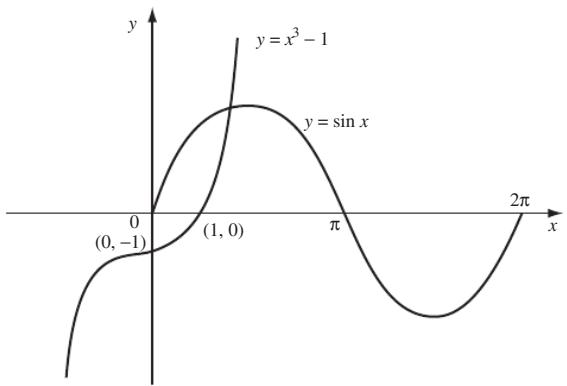
5. (a) $\sin x + \frac{\cos^2 x}{\sin x} \equiv \frac{\sin^2 x + \cos^2 x}{\sin x}$
 $= \frac{1}{\sin x} = \operatorname{cosecx}. \quad (2)$

(b) $\operatorname{cosecx} - \sin x = 3$
 $1 - \sin^2 x = 3 \sin x$
 $\sin^2 x + 3 \sin x - 1 = 0$
 $\left[\begin{array}{l} \text{OR } \frac{\cos^2 x}{\sin x} = 3 \\ 1 - \sin^2 x = 3 \sin x \end{array} \right]$

$\sin x = \frac{-3 \pm \sqrt{13}}{2}$
 $\sin x = \frac{-3 + \sqrt{13}}{2} = 0.30277\dots$

$x = 17.6^\circ, 162.4^\circ$

6.



(a) curves cross once

(b) $f(x) = \sin x - x^3 + 1$

$$\left. \begin{array}{l} f(0) = 0 - 0 + 1 \\ f\left(\frac{\pi}{2}\right) = -1.875 \end{array} \right\} \text{change in sign indicates root in interval}$$

(c) $x_1 = 1.2257,$

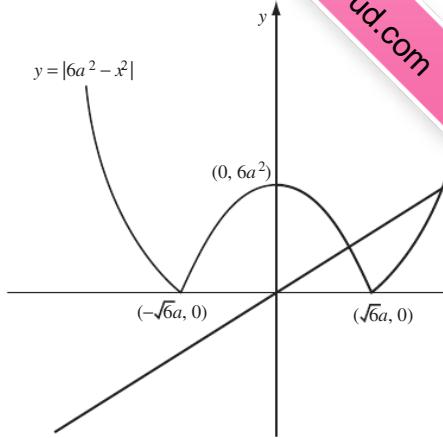
$x_2 = 1.2474,$

$x_3 = 1.2489,$

$x_4 = 1.2490,$

$\alpha = 1.2491$

7.



(4)

(1)

(2)

(3)

(a) $x = 2a, y = |6a^2 - 4a^2| = 2a^2$

$x = 2a, y = a \cdot 2a = 2a^2$

(b) $- (6a^2 - x^2) = ax$

(c) $x^2 - ax - 6a^2 = 0$

$(x + 2a)(x - 3a) = 0$

$x = -2a \text{ and } x = 3a$

second point: $x = 3a, y = 3a^2$

8. (a) $\sqrt{3} \sin x + \cos x = 2 \left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \right)$
 $R = 2, \alpha = \frac{\pi}{3}$

(b) $2 \cos \left(x - \frac{\pi}{3} \right) = \sqrt{2}$

$$\cos \left(x - \frac{\pi}{3} \right) = \frac{1}{\sqrt{2}}$$

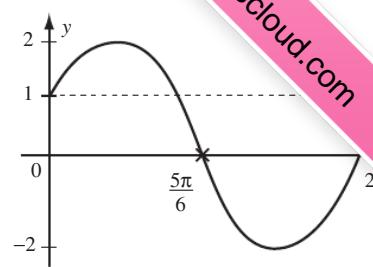
$$x - \frac{\pi}{3} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{7\pi}{4},$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12}$$

$$f(x) = 2 \cos \left(x - \frac{\pi}{3} \right)$$

(4)

(c)



(d) $2f(x) + 1 = 4 \cos \left(x - \frac{\pi}{3} \right) + 1$

maximum value = 5

minimum value = -3

(6)