Worked Solutions

Edexcel C3 Paper E

1. $\dfrac{(2x-3)\cancel{(x+1)}}{\cancel{(x+1)}} + \dfrac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}} = 3x - 5 \Rightarrow A = 3,\ B = -5$ **(4)**

$3x - 5 = x^2 - 9 \Rightarrow x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\Rightarrow x = +4 \text{ or } x = -1$ **(2)**

---

2. (*a*) $\because$ domain of $\sin^{-1}(x)$ $\quad \dfrac{-\pi}{2} < x < 0 \quad \sin x = -\dfrac{3}{5}$

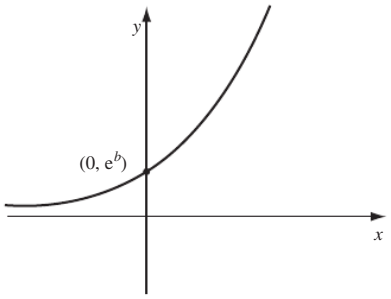
$\cos^2 x + \sin^2 x = 1$

$\therefore \cos^2 x + \dfrac{9}{25} = 1 \quad \therefore \cos^2 x = \dfrac{16}{25} \Rightarrow \cos x = +\dfrac{4}{5}$ **(4)**

(*b*) $\cos 2x = 1 - 2\sin^2 x = 1 - \dfrac{18}{25} = \dfrac{7}{25}$ **(2)**

---

3. (*a*)



(2)

(*b*) $e^b = 4 \quad \therefore b = \ln 4$ **(1)**

(*c*) $\dfrac{dy}{dx} = a \cdot e^{ax+b} = a \cdot e^{ax} \cdot e^b = 4ae^{ax}$

$x = 2, \quad 10e^5 = 4ae^{2a} \quad a = 2\dfrac{1}{2}$ **(5)**

---

4. (*a*) $\dfrac{d}{dx}\left(\cos^2 x\right) = 2\cos x(-\sin x)$

$= -2\cos x \sin x.$

(*b*) $\dfrac{d}{dx}\left(\dfrac{\ln x}{x}\right) = \dfrac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$

$= \dfrac{1 - \ln x}{x^2}$

(*c*) $\dfrac{d}{dx}\left(x^2 e^x\right) = x^2 \cdot e^x + e^x \cdot 2x$

---

5. (*a*) $\sin x + \dfrac{\cos^2 x}{\sin x} \equiv \dfrac{\sin^2 x + \cos^2 x}{\sin x}$

$= \dfrac{1}{\sin x} = \text{cosec} x.$

(*b*) $\text{cosec} x - \sin x = 3$

$1 - \sin^2 x = 3\sin x$

$\sin^2 x + 3\sin x - 1 = 0$

$\left[\begin{array}{c} \text{OR } \dfrac{\cos^2 x}{\sin x} = 3 \\ 1 - \sin^2 x = 3\sin x \end{array}\right]$

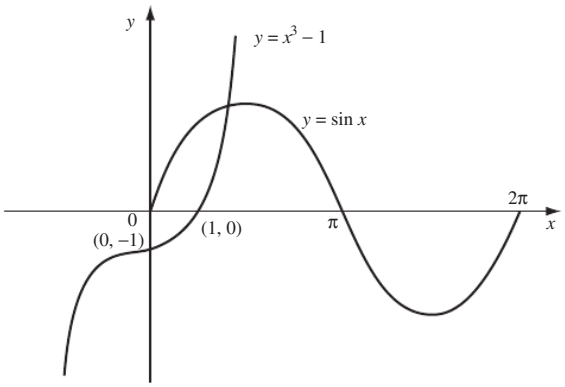
$\sin x = \dfrac{-3 \pm \sqrt{13}}{2}$

$\sin x = \dfrac{-3 + \sqrt{13}}{2}$

$= 0.30277\ldots$

$x = 17.6°,\ 162.4°$

6.



(4)

(*a*) curves cross once **(1)**

(*b*) $f(x) = \sin x - x^3 + 1$

$f(0) = 0 - 0 + 1$

$f\left(\frac{\pi}{2}\right) = -1.875$

} change in sign indicates root in interval

(2)

(*c*) $x_1 = 1.2257,$

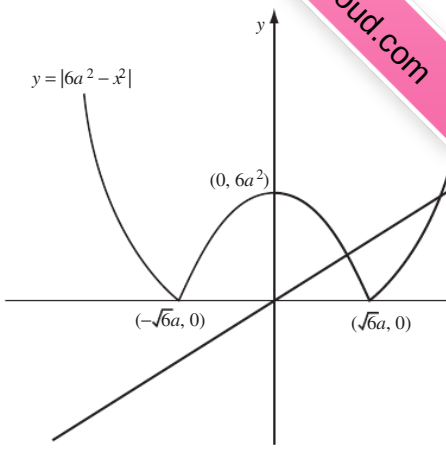
$x_2 = 1.2474,$

$x_3 = 1.2489,$

$x_4 = 1.2490,$

$\alpha = 1.2491$ **(3)**

7.



(*a*) $x = 2a,\ y = \left|6a^2 - 4a^2\right| = 2a^2$

$x = 2a,\ y = a \cdot 2a = 2a^2$

(*b*) $-\left(6a^2 - x^2\right) = ax$

(*c*) $x^2 - ax - 6a^2 = 0$

$(x + 2a)(x - 3a) = 0$

$x = -2a$ and $x = 3a$

second point: $x = 3a,\ y = 3a^2$

---

8. (*a*) $\sqrt{3}\sin x + \cos x = 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)$

$R = 2,\ \alpha = \frac{\pi}{3}$

(4)

(*b*) $2\cos\left(x - \frac{\pi}{3}\right) = \sqrt{2}$

$\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

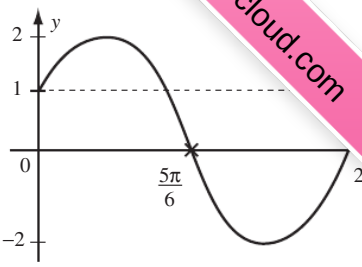
$x - \frac{\pi}{3} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{7\pi}{4},$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$

$\mathrm{f}(x) = 2\cos\left(x - \frac{\pi}{3}\right)$

(6)

(*c*)



(*d*) $2\mathrm{f}(x) + 1 = 4\cos\left(x - \frac{\pi}{3}\right) + 1$

maximum value $= 5$

minimum value $= -3$